6^{th} Lecture of Operation Research 2

Changes affecting the optimality:

2) Change in the usage coefficient (a_{ij}).

For the linear programming problem:

Max Z = $4\, X_1 + X_2 + 0\, S_1 + 0\, S_2 + 0\, S_3 + 0\, S_4$

Subject To: $X_1 \quad +2\, X_2 \leq 6$

$2\, X_1 + X_2 \ \leq 8$

$-\, X_1 + X_2 \ \leq 1$

$X_2 \ \leq 2$

$X_1 \ , X_2 \ \geq 0$

Order of basic Variable in optimal solution iteration : (S_1 , X_1 , S_3 , S_4)

$$IM = \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) If X_2 coefficients in Constraints changed from $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 4 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

Find the effect of this change on optimal Solution.

$$(Y_1 \ \ Y_2 \ \ Y_3 \ \ Y_4) = (0 \quad 4 \quad 0 \quad 0) \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = (0 \quad 2 \quad 0 \quad 0)$$

Coeff. Of X_2 in z-row:

$$4\, Y_1 + 3\, Y_2 + Y_3 + Y_4 - 1 = 0 + 3 * 2 + 0 + 0 - 1 = 5$$

Then the solution Still optimal.

$X_1^* = 4 \qquad X_2^* = 0 \qquad Z^* = 16$

b) If X_2 coefficients in Constraints changed from $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

Coeff. Of X_2 in z-row:

$$1\,Y_1 + 0\,Y_2 + Y_3 + Y_4 - 1 = 0 + 0 + 0 + 0 - 1 = -1$$

Then the solution is not optimal.

الحل مش Optimal يبقى هبدأ أجيب عمود الـ X_2 الجديد واحطه فى الجدول واشتغل Primary Simplex علشان أجيب الـ Optimal Solution .

$$\begin{bmatrix} New \\ X_2 \\ col \\ umn \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Basic	X_1	X_2	S_1	S_2	S_3	S_4	Sol.
Z	0	-1	0	2	0	0	16
S_1	0	1	1	-1/2	0	0	2
X_1	1	0	0	½	0	0	4
S_3	0	1	0	½	1	0	5
S_4	0	1	0	0	0	1	2
Z	0	0	1	3/2	0	0	18
X_2	0	1	1	-1/2	0	0	2
X_1	1	0	0	½	0	0	4
S_3	0	0	-1	0	1	0	3
S_4	0	0	-1	½	0	1	0

$X_1^* = 4$ $\quad\quad$ $X_2^* = 2$ $\quad\quad$ $Z^* = 18$

3) Addition of a new activity.

For the following Linear Programming problem:

Max Z = 3 X_1+2 X_2 + 3/2 X_3+ 0 S_1 + 0 S_2 + 0 S_3 + 0 S_4

Subject To: X_1 +2 X_2 + 3/4 $X_3 \leq 6$

2 X_1+ X_2 + 3/4 $X_3 \leq 8$

- X_1+ X_2 − $X_3 \leq 1$

$X_2 \leq 2$

X_1 , $X_2 \geq 0$

Given That (Y_1 Y_2 Y_3 Y_4) = (1/3 , 4/3 , 0 , 0) , IM = $\begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix}$

هبدأ اجيب معامل الـ X3 اللى أنا ضفيته علشان اشوف الحل لسه Optimal ولا لاء

X_3 Coeff. In Z-row:

$$3/4\,Y_1 + 3/4\,Y_2 - Y_3 - 3/2 = -1/4$$

My problem is Max and X_3 Coeff. Is negative Then solution is not optimal.

هنضطر نجيب عمود الـ X_3 الجديد ونحل المسالة Primary Simplex علشان نجيب الـ Optimal solution

$$\begin{bmatrix} New \\ X_3 \\ \\ column \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} ¾ \\ 3/4 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} ¼ \\ 1/4 \\ -1 \\ -1/4 \end{bmatrix}$$

هجيب عمود الـ X_3 اللى انا طلعته و هحطه فى الجدول وابدأ أحل Primary Simplex :

Basic	X1	X2	X3	S1	S2	S3	S4	Sol.
Z	0	0	-1/4	1/3	4/3	0	0	38/3
X2	0	1	1/4	2/3	-1/3	0	0	4/3
X1	1	0	1/4	-1/3	2/3	0	0	10/3
S3	0	0	-1	-1	1	1	0	3
S4	0	0	-1/4	-2/3	1/3	0	1	2/3
Z	0	1	0	1	1	0	0	14
X3	0	4	1	8/3	-4/3	0	0	16/3
X1	1	-1	0	-1	1	0	0	2
S3	0	4	0	5/3	-1/3	1	0	25/3
S4	0	1	0	0	0	0	1	2

$X_1^* = 2$ $\qquad X_3^* = 16/3$ $\qquad Z^* = 14$

Best Wishes